

DETERMINATION OF GEOMETRIC HABITS OF HIGH SCHOOL STUDENTS BY ORIGAMI ACTIVITY

Origami Etkinlikleri İle Öğrencilerin Geometrik Zihin Alışkanlıklarının Belirlenmesi

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ABSTRACT

The aim of this research is to determine the geometric habits of high school students by using origami. During the study, formation of object and activities with origami were planned and implemented. The activities were held in the mathematics lesson related to geometry and mathematics working groups. The competencies of students to reveal their geometric habits were determined during these activities. "Geometric mind habits worksheets" prepared by the researcher were also applied to determine the competencies. In the study, students' geometric thinking processes were examined according to four components. These components have been identified by Driscoll, DiMatteo, Nikula, and Egan (2007) as "associating by reasoning, examining invariants, generalizing geometric ideas and balancing discoveries with reflections". The study group of the study consisted of the students studying in the tenth grade students (NS10 = 20) and 11th grade students (NS11 = 20) studying in the center of Balıkesir. It is concluded that the average of geometric mind habits is slightly above the average. Research can be developed through studies that include cognitive and affective dimensions.

Key words: geometry habits of mind, origami, high school students, spatial thinking, geometry

ÖZET

Bu araştırmanın amacı lise öğrencilerinin geometrik alışkanlıklarını origami kullanarak belirlemektir. Çalışmada origami ve cisim oluşturma etkinlikleri planlandı ve uygulandı. Çalışma gruplarına etkinlikler geometri ve matematik konularını içeren matematik dersinde gerçekleştirildi. Bu etkinlikler sırasında öğrencilerin geometrik alışkanlıklarını ortaya koyma yeterlilikleri belirlendi. Yetkinlikleri belirlemek için araştırmacı tarafından hazırlanan "geometrik zihin alışkanlıkları çalışma yaprakları" uygulanmıştır. Araştırmada öğrencilerin geometrik düşünme süreçleri dört bileşene göre incelenmiştir. Bu bileşenler, Driscoll, DiMatteo, Nikula ve Egan (2007) tarafından "akıl yürütme ile ilişkilendirme, değişmezleri inceleme, geometrik fikirleri genelleme ve keşifleri yansımalarla dengeleme" olarak tanımlanmıştır. Araştırmanın çalışma grubunu Balıkesir merkezinde bulunan onuncu sınıfta (NS10 = 20) okuyan öğrenciler ve 11. sınıfta okuyan öğrenciler (NS11 = 20) oluşturmaktadır. Çalışmada öğrencilerin geometrik zihin alışkanlıklarının ortalamasının biraz üzerinde olduğu sonucuna varılmıştır. Ayrıca çalışmada, origami kullanarak geometride bir soruyu çözmek için iki ve daha fazla alışkanlığın kullanıldığı sonucuna varılmıştır. Araştırma, bilişsel ve duyuşsal boyutları içeren çalışmalar ile geliştirilebilir.

Anahtar Kelimeler: zihnin geometrik alışkanlıkları, origami, lise öğrencileri, uzamsal düşünme, geometri

1. INTRODUCTION

Learning mathematics is not limited to the acquisition of basic concepts and skills. It includes mathematical thinking (Arslan&Yıldız, 2010; Yıldız, 2016), reasoning (MoNE, 2013), understanding problem solving strategies (Altun, 2015) and realizing the mathematics used in real life. It has been observed that paper folding method has an impact on numbers-operations (Akan-Sağsöz, 2008) geometry (Arıcı, 2012; Akayuure, Asiedu-Addo, 2011) and algebra (Georgeson, 2011; Higginson & Colgan, 2001), learning (Georgeson, 2001); proving skills of students (Georgeson, 2011); spatial thinking (Çakmak, 2009) and seeing spatial relationships (Akayuure, Asiedu-Addo, 2011). Our curriculum gives students concrete experiences and emphasizes their abstract thinking and making connections (Ministry of National Education-MoNE, 2013). For this reason, it is important to have applications that enable students to see mathematics as "felt, worthwhile, and important" and to work with "care and patience" (MoNE, 2013).

Geometry, which is one of the basic learning areas of mathematics, has an important place in the development of mathematical thinking skills. The geometry learning area has an important place in the mathematics program and also helps to understand both the mathematics course and other courses in the curriculum (Van de Walle, Karp & Bay-Williams, 2014). Van Hiele (1999) and Van

de Walle, Karp and Bay-Williams (2007) defined geometric thinking as the development of spatial ability as a way of thinking that helps individuals overcome geometry problems by establishing geometric relationships between objects. Spatial ability has been defined as being able to create a visual image, maintain a shape, rearrange it and transform it into another shape (Lohman, 1993). Olkun and Altun (2003) have defined spatial relationships as the student's ability to turn over two and three-dimensional geometric forms in his/her mind as a whole, and to recognize them in their various positions, and on the other hand, spatial visualization as the ability to visualize new situations that will occur as a result of moving two and three dimensional objects consisting of one or more pieces and images of their parts in three-dimensional space. Geometric thinking levels are of great importance in determining spatial abilities. Improved geometric thinking levels of individuals will enable them to acquire 21st century skills such as problem solving, critical thinking, and understanding the concept of dimension. Therefore, it is necessary to determine how students think, how they infer, and what strategies they use to develop geometric thinking.

When national and international exams are examined, it is seen that the rate of solving geometry questions is well below the average (The Measuring, Selection and Placement Center (MSPC), 2015, 2016, 2017, 2018, 2019). The importance of geometry in students' lives is undeniable, but students' achievements in geometry are very low in both national and international exams. According to the PISA exam, the average mathematics literacy of Turkish students is at the 2nd level and below. According to the PISA assessment, students who have reached level 2 can only recognize and interpret situations where no skill other than direct inference is required. These students can obtain the necessary information from a single source and use only one form of notation. Students at this level can use basic algorithms, formulas, and the usual ways of processing. They can do simple reasoning such as direct proof and make comments that do not go beyond what is seen on the results. With regard to high school curriculum, this is incompatible with the goals of higher-order thinking skills, such as using synthetic, analytical and vectorial approaches in geometric proofs, questioning the validity of inferences and generalizations, and transforming geometric objects into algebraic objects. In addition, according to the 2009 Report of Determining Student Achievement Exam, the mathematics success of 9th and 10th grade high school students is 32% and 36%, respectively. According to the results of the survey conducted among students, the most difficult course of the students is geometry. Due to the low geometry success of both secondary and high school students, urgent and effective measures should be taken. Olkun and Aydoğdu (2003) emphasized that students' failure may be due to the fact that teachers encouraged students to memorize geometric shapes and concepts.

Mental habits are the ability to choose and apply appropriate ones among the high-level mental skills unique to humans (Leikin, 2007). The mathematical habits of the mind can be expressed as thinking about mathematical concepts and reaching mathematical problems in special ways. The mathematical habits of the mind cover intellectual activities such as predicting, questioning solutions, exploring patterns, customizing, classifying, analyzing and proving problems and methods by using alternative representation (Goldenberg, Shteingold & Feurzeig, 2003). The concept of "mind habits" by researchers has been tried to be introduced with many skills. It is the "mental habits" skills list of Costa and Kallick (2000), which are the most accepted of these skill lists. This list includes 16 skills. These are as follows: persistence, acting by thinking, listening others with insight and empathy, flexible thinking, metacognition, striving for accuracy and certainty, asking questions and posing problems, applying previous knowledge to new situations, thinking with clarity and certainty and communicating, collecting data with all senses, creating, dreaming and innovating, responding with confusion with wonder and fear, taking risks with responsibility, finding humor, interconnected thinking and continuous learning. Mental habits (Cuoco, Goldenberg & Mark, 1996) are handled in two forms as;

- ✓ General mental habits and
- ✓ Discipline-specific mental habits.

General mental habits include skills such as thinking, researching, recognizing patterns and relationships, making definitions, discovering, making assumptions, and visualizing. Marc, Cuoco, Goldenberg, and Sword (2010) express the mathematical habits of the mind as always having the ability to reason by considering the methods used by those interested in mathematics and by abstracting them as they do through intellectual activities in the face of unusual situations. The development process of mental habits includes;

- ✓ Exploring mathematical ideas,
- ✓ Formulating the problem situation
- ✓ Configuration of examples
- ✓ Developing a problem-solving approach that can be useful in similar problem situations
- ✓ More detailed screening of an expression that can generalize the mathematical situation they are working on
- ✓ Making checks to determine if there is an error in their solution (Jacobbe, 2007).

In the Secondary School Mathematics Lesson (Grades 5-8) Curriculum (Ministry of National Education (MoNE), 2013), mathematical practice habits within the subheading of “Reasoning” under the main title of “Mathematical Process Skills” (MoNE, 2013, 5) are given as;

- ✓ “Defending the accuracy and validity of the inferences,
- ✓ Making logical generalizations and inferences,
- ✓ Explaining and using mathematical patterns and relationships while analyzing a mathematical situation,
- ✓ Making predictions about the outcome of transactions and measurements using strategies such as rounding, grouping the appropriate numbers, using the first or last digits, or strategies they have developed themselves.
- ✓ Making an estimate of measurement taking into account a particular reference point”

Obviously, one of the goals of our curriculum can be stated that students are aimed to gain mathematical mind habits.

1.1. The aim of the research

The aim of the research is to determine the geometric habits of high school students using origami activities. In this context, the research will proceed in three dimensions. Firstly; designing origami activities and worksheets to observe their ability to reveal “geometrical habits of the mind”; and secondly; determining of the “geometric habits of the mind” of high school students with the designed activities and worksheets.

1.2. The importance of the research

In mathematics tests conducted by Trends in International Mathematics and Science Study (TIMSS) applied to eighth grade students of various countries, Turkey ranks 31st among 38 countries in the exam held in 1999 and ranks 30th among 59 countries in the exam conducted in 2007 (Department of Education Research and Development, 2003; 2011). Looking at the analysis of the questions in geometry and the results involving the use of geometric reasoning skills, it has been highlighted that the students in Turkey had difficulty in terms of the use of these skills and this situation made it necessary to review geometry education in our country. The geometric habits that students have are very important as they also affect students' thinking skills. Indeed, since these habits meet the geometry of the student at a young age and are effective for life, they will contribute to the research area. Due to the fact that the number of the studies through the Geometric Habits of the Mind model by Driscoll, DiMatteo, Nikula and Egan (2007), and the studies in the literature based on the development of the mind's geometrical habits processes and exemplifying the teaching process is low in number and those studies are mostly for teachers and prospective teachers, there is not a clear picture of how high school students' geometric habits processes of the mind are realized. For this reason, by using origami, students will develop their spatial skills and develop as they make

folds and make inquiries in some geometric concept definitions (such as concave, convex, equilateral triangle, rectangle, deltoid, trapezoid) that they have difficulties at the beginning, and they are expected to do both mathematical inquiries and inquiries about transformation of geometric concepts. It is suggested that the origami lessons are not just a paper-folding process, but also, emphasis is placed on the importance of using them as a powerful tool to develop students' geometric thinking habits.

1.3. The Original Value

When the literature is examined, it is seen that there are many studies which reveal that students do not use the geometrical habits of the mind at the expected level (Cuoco, Goldenberg & Mark, 1996; Dostal, 2000; Fenderson, 2010; Gordon, 2011; Kılıç, 2013, Köse & Tanisli, 2014; Lim & Selden, 2009; Matsuura, Sword, Beth-Piecham, Stevens & Cuoco, 2013). On the other hand, in some studies it has been stated that students' geometrical habits of the mind can be developed (Cuoco, Goldenberg & Mark, 1996; Gordon, 2011; Matsuura, Sword, Beth-Piecham, Stevens & Cuoco, 2013). As can be seen, it is important to design different learning environments that include the use of origami, which has the potential to develop geometrical habits of the mind, and to determine how students are affected by these teaching habits.

2. METHOD

2.1. Model of the Research

A case study pattern, one of the qualitative research methods, will be used in the research. Yin (2009) defined the case study as a research pattern that works in an actual case within its real-life environment, where the boundaries between the case and the environment in which it is located are not clearly defined and where more than one source of evidence or data is available.

2.2. Participants

The sample of the study consisted of 10th and 11th grade students ($N_{S10} = 20$ and $N_{S11} = 20$) studying in the spring semester of 2017-2018 and fall 2018-2019. The study is limited to the students of a high school in the West Marmara Region. The participants' high scores of High School Entrance Exam, the equal success rates and being able to do the geometry questions played a role in their selection. The reason for the selection of the 10th and 11th grade students is for the fact that the 9th grade has passed the adaptation phase and is not concerned about the preparation for 12th grade Field-Proficiency Test and Basic Proficiency Test. Students voluntarily participated in the study and the names of the participants were not included in the research. The findings of the research are also limited to the qualitative analysis processes presented in the method section. It is the most suitable method for making deep analysis.

2.3. The Scope and Process of the Study

In this study, it was aimed to determine mental geometric habits of science high school 10th and 11th grade students while proving theorems with classical origami and modular origami and creating platonic objects. Science high school students were selected in order to obtain rich data. The 10th and 11th grades were chosen because they started to study the geometric shapes and features from the tenth grade. In order to examine students' mental habits, it was aimed to use origami to divide an angle of the square or rectangle into three, to make the largest equilateral triangle from one edge, to form an object using square papers, to reach generalization by making tetrahedron, hexahedron, octahedron, dodecahedron, icosahedron, to create cubes from square, to create cubes using modular origami, and to question each folding step while doing these. The research was carried out for six weeks, two hours a week. In the first week, students were studied about the basic folding types, the history of origami, the use of origami today, simple origami animal and plant studies.

2.4. Research Questions

The subject of this research is to investigate the geometric habits of the mind that high school students exhibit while forming their geometric habits with origami. For this purpose, the study question of the research is: What are the geometrical habits of the mind that tenth and eleventh grade students exhibit when they create their geometric habits with origami?

2.4.1. Sub Research Questions

How should it be to design origami activities and worksheets to observe the competencies of revealing the geometric habits of the mind?; How do the designed activities and worksheets affect High School students' processes of geometric habits of the mind including reasoning with relationships; investigating invariants, generalizing geometric ideas, and balancing exploration with reflection?

3. LITERATURE

3.1 Mind Habits

When studies conducted in recent years in mathematics education are examined, it is seen that studies on "Mathematics and Mind" have increased. The mathematical habits of the mind, on the other hand, are expressed as having the ability to reason, through intellectual activities, against different unusual situations by considering the methods used by those engaged in mathematical science and by abstracting them in the way they do (Mark, Cuoco, Goldenberg & Sword, 2010). In the literature, problem-solving strategies such as generalization, being systematic and organized, predicting answers and checking solutions, solving simple-related problems, searching patterns and regularities, justifying the solution stand out as mathematical practice habits related to thinking within the mathematical habits of the mind (Goldenberg et al., 2003). Although there are studies on mind habits in the international literature (Gordon, 2011; Mark et al., 2010; Matsuura et al., 2013), it is seen that almost all of these studies are aimed at teaching high school mathematics or making conceptual explanations. In the national literature, a few studies on the subject of mind habits have been carried out (Tümüšoğlu, 2013; Köse & Tanisli, 2014; Kilic, 2013; Boz 2015; Boz-Yaman 2015, 2017; Bozkurt & Koc 2016; Duatepe-Paksu & Boz-Yaman 2017; Ersen 2017; Gürbüz & Güler, 2018) and it is seen that the studies on this subject have increased in recent years.

Emphasis is placed on the importance of individuals' problem-solving skills in mathematics education programs (Gordon, 2011). Driscoll et al. (2007) defined the Geometric Habits of the Mind (GHM) in order to increase students' geometric thinking skills in terms of geometric thinking habits, and explained how these habits should be understood by teachers and how they would contribute to students' geometric thinking. Driscoll et al. (2007) say that one should acquire some mental habits in order to develop geometric thinking and define these habits as the geometric habits of mind. The geometric habits of the mind have 4 subcomponents (Bozkurt & Koç, 2016; Driscoll et al., 2007). These are as follows (Figure 1):

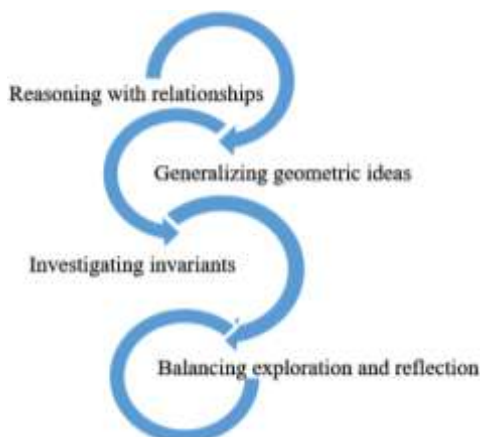


Figure1. The Geometric Habits of the Mind

Reasoning with relationships

Reasoning habit includes looking for relationships (such as parity, similarity or parallelism) between one, two or three-dimensional geometric shapes and objects and being able to reason how these relationships can assist in problem-solving processes. Individuals who have this habit; determine the common/similar or non-common properties between two or more geometric shapes and the situations that fit the definition or not. They can reveal the similarities or differences between these figures for their reasons. They can examine the symmetry in a given geometric shape and its relation with other shapes.

b. Investigating invariants

The habit of investigating invariants involves analyzing which properties of a geometric shape as a result of a transformation (reflection, translation, rotation, disassembly, enlarging the shape, or controlled shape changing) remains the same, which properties changes between the first state and the last state, whether there is an inverse example and whether generalization is reached or not.

c. Generalizing geometric ideas

The process of generalizing geometric ideas is the stage that includes the general and all situations where geometric facts are understood and defined. It is investigated whether the property of a geometric shape or object is in the whole of that shape or object. Individuals with this habit can consider specific situations on the subject, experiment for different examples after special situations, and then make generalizations for new situations. They can see the entire solution set and explain why there are no other solutions. In this process stages, results and properties of geometric shapes are generalized.

d. Balancing exploration through reflection

Driscoll defines balancing exploration and reflection, which is the last habit, as being aware of what the person is doing when solving a problem, and questioning the accuracy of the results he finds at each stage of the solution and after the solution. These defined habits are not independent habits, but rather interconnected habits needed in the problem-solving process (Driscoll, 2007). It aims the student to discover and learn. The process of balancing exploration and reflection is the stage in which the part up to the point of arrival is evaluated by using different approaches in solving the problem. Individuals who have the habit of exploring and reflecting can draw, play, or make discoveries through guesses or intuition. In each step in the solution process, they question themselves about the result. They can identify intermediate steps well for solution (Driscoll et al., 2007; Driscoll et al., 2008). Considering all four components, origami is considered to be the most effective way to develop students' mental geometric habits.

3.2. Origami

The term paper folding is made up of the Japanese words “oru” which means folding or “kami” which means “paper” (Gür, 2015). Origami can be classified as “classical origami”, “modular origami”, and “wet origami”. In classical origami, various animal or article figures are made from one piece of paper, while in modular origami, similar geometric pieces can be combined to create three-dimensional geometric models (Tuğrul & Kavici, 2002). With paper folding, various figures can be obtained using geometric shapes (Krier, 2007). As indicated by Tuğrul and Kavici, 2002; Boakes, 2006; Golan and Jackson, 2010 and Arslan et al., 2013 origami is suitable for use as a method in education since it covers all visual, affective, auditory and kinesthetic skills.

Secondary education and higher education mathematics are among the application areas of paper folding method and mathematics teaching. How to obtain a box with the largest volume from a paper can be demonstrated using the paper folding method, but to find the largest volume of the box, “Analysis” knowledge is required (Wares, 2011). It is thought that classroom practices using paper folding method in mathematics lessons will be useful. In addition, this method will help

students to have positive attitudes towards mathematics by preventing their prejudices towards mathematics lessons (Boakes, 2009; Çakmak, 2009; Tuğrul & Kavici, 2002). In summary; in addition to gaining psychomotor and affective skills, the activities and models will greatly assist in the development of mathematical process skills such as problem solving (Coad, 2006; Mastin, 2007) and communication-reasoning-relating. Gürbüz and Güler (2018), in their study titled “Investigation of Geometric Habits of the Mind Using Paper Folding” showed that paper supports geometric habits of the mind, students can realize that they can reach solutions by embodying intangible questions through paper folding and they can realize that the main components of triangles do not change.

3.3. Data Collection Tool

In order to determine the geometric thinking habits of 10th grade and 11th grade science high school students, the Geometric Thinking Habits Worksheets, which consisted of questions that were published by Ministry of National Education, geometry and origami in the literature, were applied by the researcher (Appendix-1). A worksheet was consisting of six parts namely A B, C, D, E, F worksheets. The students were done concrete materials. The worksheets and materials were used as data collection tools and were examined through the rubric provided in the evaluation of the data.

A worksheet: Drawing of regular polyhedrons to determine students’ spatial abilities and forming concrete polyhedrons, reaching the generalization of polyhedrons worksheet designed to create mathematical process

B Worksheet: Finding correlation between the diagonal length of the rectangle and the edges by folding the origami worksheet

C Worksheet: Generalizing from the surface, crown and edge numbers of polyhedron, concavo-convex worksheet

D worksheet: Dividing an angle into three equal parts worksheets

E worksheet: Creating a 60° angle in rectangular with origami worksheet

F worksheet: Creating polyhedron using a classic and modular origami worksheet

The worksheet in A includes the habits of both *reasoning with relationships and investigating invariants* (being able to name with edges, surface numbers and details namely invariants). The worksheet in B includes the habits of *reasoning with relationships as well as investigating invariants and exploration and reflection* (using the lengths of special triangles and special triangles, calculating the hypotenuse and diagonal length, and identical triangles). The worksheet in C1 and C2 covers the habits of both *reasoning and investigating invariants, reaching generalization* (includes the ability to find edges, surface numbers and edge numbers, and name with invariants; to explore the relationship between special quadrilaterals and shapes). The worksheet in D includes the habits of *exploration and reflection, then reasoning, researching invariants and the generalization of geometric ideas* (dividing the square into an equal rectangle and dividing it again into an equal rectangle, using parallelism, making additional drawings, seeing relationships, equality of rectangles, seeing trapezoidal triangle properties, equation of triangles, dividing the angle into three). The worksheet in E includes the use of the habits of *reasoning and investigating invariants habits after exploration and reflection* (steepness, middle strut, equilateral triangle, similarity in the triangle, parallelism, additional drawing, reaching the triangle formed by angles of 90° and 30°). The worksheet in F includes the habits of *balancing exploration and reflection, reasoning with relationships, investigating invariants and the generalization of geometric ideas* (Creating polyhedrons using classical and modular origami: folding using square paper, obtaining equal areas, using equilateral triangles and properties, creating objects with folds). By creating tetrahedron, octahedron, icosahedron through the use of origami, students are provided with the habits of *exploration and reflection, reasoning with relationships and generalization of geometric ideas*. In order to ensure the validity of the geometric thinking habits

worksheets, a pilot study was conducted on the problem of whether the problems in the measuring tool represent the area to be measured. 10 students who were not included in the main research process were asked to solve the questions on the worksheet and discussions were held on how they solved them.

Thus, it was checked whether the problems are compatible with the geometric thinking habits indicators determined by the researcher. Then, the opinions of 3 specialist mathematics educators and 3 high school math teachers for worksheets were taken and the language, level, content and scope validity of the worksheets of geometric thinking habits consisting of open-ended problems were provided. In addition, for the analysis of these qualitative data, the compatibility between the coders was calculated and the agreement was found to be 84%. Full coordination was achieved at the end of consultations with coders. The formula developed by Miles and Huberman (1994: 100) was used for the reliability of the study. $Reliability = \frac{Consensus}{Consensus+Disagreement}$. Since this rate is over 70%, the study is reliable (Miles & Huberman, 1994).

3.4. Data Analysis

Descriptive and content analysis will be used in the analysis of qualitative data. Content analysis was applied to student worksheets. Cohen, Manion and Morrison (2007) content analysis has been preferred because it combines similar data within certain concepts and themes and transforms this data in a way that the reader can make sense of (Yıldırım & Şimşek, 2005). In the research process, content analysis was conducted and interpreted by taking into account students' operations, explanations, the shapes they drew, the concrete proofs they made and the objects they created. Names of students were not included. Based on the emerging comments, their geometric habits were examined. Students' explanations were handled in terms of the geometric habitual processes of the mind and the data were coded. The answers given by students to open-ended questions were presented in the findings part in details. Using the clinical interview data, concrete materials and proofs of the students, the form of "the components of examining students' geometric thinking processes" developed by Driscoll et al. (2007), namely, the form of "geometric habits of the mind" was used (Table 1).

Table 1. Rubric: Students' Mental Habits Processes

Mental Habits	Codes
Reasoning with relationships	Recognition (knowing the properties of geometric shapes, associating different geometric shapes with each other), proportional reasoning (accompaniment and similarity), using symmetry. Reasoning habit includes looking for relationships (such as parity, similarity or parallelism) between one, two or three-dimensional geometric shapes and objects and being able to reason how these relationships can assist in problem-solving processes
Investigating invariants	Even if some features change, noticing the features that always remain constant, thinking of extreme situations. For example: finding the diagonal length in special triangles.
Generalizing geometric ideas	Finding all possible situations, finding the unknown from what is known, acting from a special situation and adapting it to the general.
Balancing exploration through reflection	Being able to follow a controlled discovery process, to develop different solution strategies, and to make consistent explanations about the correctness of the problem solution. For example: Finding length using theorem proof by folding the paper through Origami

In the coding phase, as shown in Table 1, descriptive codes were created for these habits. Additionally, if no thinking habits were used or left blank, 0 points; 1 point if s/he used only one habit; if more than one habit was used and no solution was reached, 2 points were given, and if one /more geometric thinking habits were used and the correct result was achieved, 3 points were given. The analysis was based on the geometric thinking habits put forward by Driscoll et al. (2007). In this context, gains in the field of geometry and measurement and learning were coded according to the habits of reasoning with relationships, generalizing ideas, investigating invariants and balancing exploration and reflection.

4. FINDINGS AND COMMENTS

Findings of the first research question: "origami activities to revealing the geometric habits of the mind": Geometric Habits of the Mind processes are as follows: "Reasoning with Relationships"; "Investigating invariants", "Generalizing Geometric Ideas", "Balancing Exploration with reflection". Some of the findings are given of geometric habits below. On the left of the tables are the answers of the 10th grade student and the ones on the right are the answers of the 11th grade students (Table 2 to Table 15).

Table 2. The specification of polyhedrons

10 th class students	11 th class students

Their habits include the habits of: "1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants" (Table 2). When the student's work is examined for Figure II, it is seen that his/her comment is not correct. S/he said that the base is hexagon and there are 5 side areas and it is hexahedron. The 11th grade student did not make such a mistake. 75% of 10th Grade and 80% of 11th Grade got 3 points from this question.

Table 3. Calculation of B'D' length on diagonal (using origami)

10 th class students	11 th class students

Their habits include the habits of: “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. In both groups, most of the students reached the correct conclusion. 60% of 10th grade and 70% of 11th grade received 3 points from this question (Table 3).

Table 4. C1

10 th class students	11 th class students

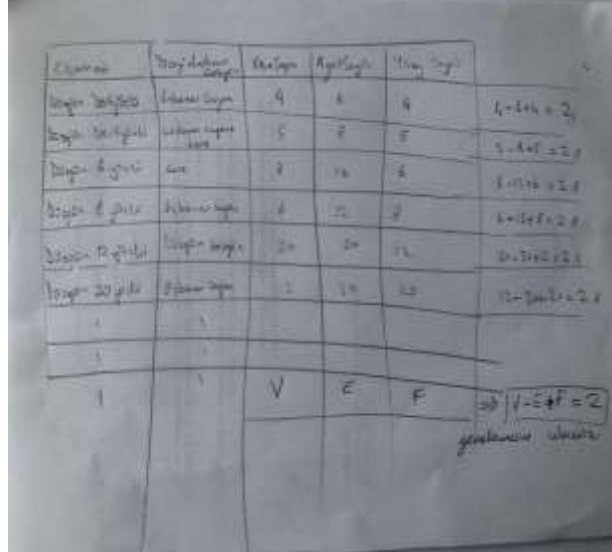
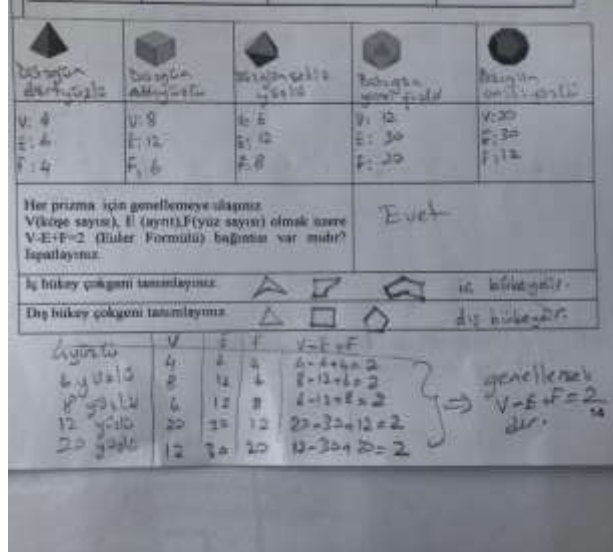
Their habits include the habits of: “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. In both groups, most of the students had problems in concave and convex definition. However, it is not possible to say the same for concave and convex definitions (Table 4). The 11th grade student showed only concave and convex by drawing shapes, but no definition. 60% of Grade 10 and Grade 11 received 3 points from this question.

Table 5. C1: Corner, Edge, Surface Number and Kepler Polyhedrons of Polyhedrons

10 th class students	11 th class students

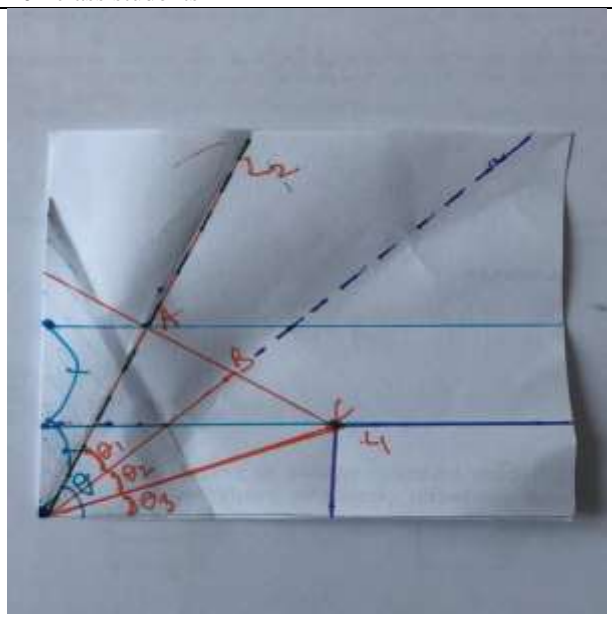

Their habits include the habits of: “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. In both groups, it is seen that most of the students reached the generalization (Table 5). 10th grade and 11th grade received 3 points from this question.

Table 6. Reaching Generalization

10 th class students						11 th class students					
											

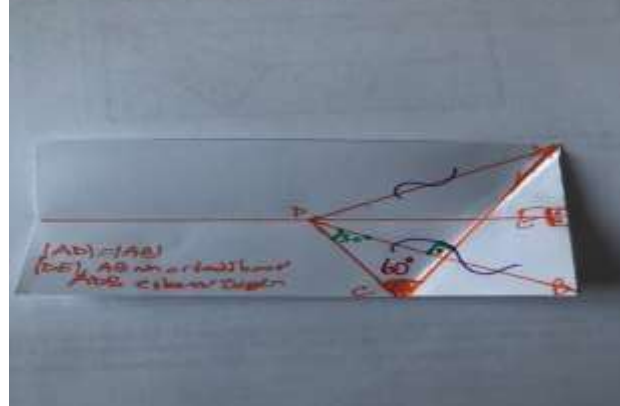
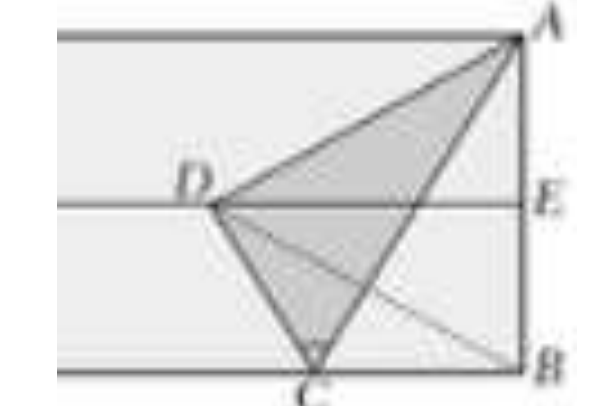
Their habits include the habits of: “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. In both groups, it is seen that most of the students reached the generalization (Table 6).

Table 7. Dividing an angle into three with origami

10 th class students		11 th class students	
			



Their habits include the habits of: “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. It is seen that in both groups, most of the students have reached the correct result. However, mistakes are seen in the drawing and achievement of 10th grade students (Table 7). Only 50% of Grade 10 and Grade 11 received 3 points from this question.

Table 8. Creating 60° angle with origami

10 th class students	11 th class students
	



Their habits include the habits of: “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. It is seen that in both groups, most of the students have reached the correct result (Table 8). 50% of Grade 10 and 60% of Grade 11 received 3 points from this question.

Table 9. Creating right prism with origami

10 th class students	11 th class students
	

Includes the habits of “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. It is seen that in both groups, most of the students have reached the correct result (Table 9). 40% of the 10th grades and 35% of the 11th grades received 3 points from this question.



Table 10. Creating cube with classic and modular origami

10 th class students	11 th class students
	

Includes the habits of “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. It is seen that in both groups, most of the


students have reached the correct result (Table 10). 40% of 10th Grades and 35% of 11th Grades received 3 points from this question.

Table 11. Creating cube with modular origami

10 th class students	11 th class students
	

Includes the habits of “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. It is seen that in both groups, most of the students have reached the correct result (Table 11). 40% of 10th Grades and 35% of 11th Grades received 3 points from this question.

Table 12. Creating icosahedron with modular krigami (20 faces-icosahedron)

10 th class students	11 th class students
	There are no example.

Includes the habits of “1. Balancing exploration and reflection 2. Reasoning with relationships 3. Generalizing geometric ideas 4. Investigating invariants”. It is seen that in both groups, most of the students have reached the correct result (Table 12). 40% of 10th Grades and 35% of 11th Grades received 3 points from this question.

4.1. Descriptive Analysis of Scores from Geometric Thinking Habits Worksheets

Descriptive statistics for each problem in the Geometric Thinking Habits are included in Table 13 and Table 14. The rubric prepared was used for scoring. (0 points: any thinking habit was used or left blank; 1 point: one habit was use; 2 points: more than one habits were used but unreached solution, and 3 points: one/more thinking habits were used and reached correct solution). Descriptive statistics of the 10th grade science high school students who participated in the study are given in Table 13.

Table 13. Descriptive analysis of the scores from the questions in the 10th grade Geometric mind habits

Questions	SCORES							
	0 point		1 point		2 points		3 points	
	f	%	f	%	f	%	f	%
Worksheet A	1	5	1	5	3	15	15	75
Worksheet B	1	5	3	15	4	20	12	60
Worksheet C1	1	5	2	10	3	15	14	70
Worksheet C2	2	10	2	10	4	20	12	60
Worksheet D	2	10	3	15	5	25	10	50
Worksheet E	2	10	2	10	6	30	10	50
Worksheet F	3	15	3	15	6	30	8	40

The number of people who got 0 points and 1 point in the solution of Worksheet A is five percent. 75% of the participants used two or more geometric thinking habits. On B and C2 worksheets 60% of the participants used two or more geometric thinking habits; on D and E worksheets 50% of the participants used two or more geometric thinking habits and on F worksheets only 40% of the participants used two or more geometric thinking habits in order to reach the conclusion.

Worksheet A average score is $\bar{x} = 2,65$

Worksheet A average score is $\bar{x} = 2,40$

Worksheet A average score is $\bar{x} = 2,55$

Worksheet A average score is $\bar{x} = 2,40$

Worksheet A average score is $\bar{x} = 2,25$

Worksheet A average score is $\bar{x} = 2,30$

Worksheet A average score is $\bar{x} = 2,10$ and Grade 10 overall worksheet average score is $\bar{x} = 2,37$ ($2,37 \times 7 = 16,59$).

When the data in Table 13 are analyzed, it is seen that the average scores of the students in the 10th are $\bar{x} = 16.59$. From here, it can be said that the achievements of students' geometric thinking habits are at a medium level.

Tablo 14. Descriptive analysis of the scores from the questions in the 11th grade Geometric mind habits

Questions	SCORES							
	0 point		1 point		2 points		3 points	
	f	%	f	%	f	%	f	%
Worksheet A	-	-	2	10	2	10	16	80
Worksheet B	1	5	2	10	3	15	14	70
Worksheet C1	1	5	2	10	3	15	14	70
Worksheet C2	1	5	3	15	4	20	12	60
Worksheet D	2	10	4	20	4	20	10	50
Worksheet E	1	5	3	15	4	20	12	60
Worksheet F	3	15	4	20	6	30	7	35

Participants solved the worksheet A by including two or more geometric thinking habits with the highest percentage of 80%. In C1, C2 and D worksheets the percentage is the same for 3 points in both Grade 10 and 11 grades. In the F worksheet, the percentage is very low for 3 points at both grade levels.

Worksheet A average score is $\bar{x} = 2,70$

Worksheet A average score is $\bar{x} = 2,55$

Worksheet A average score is $\bar{x} = 2,55$

Worksheet A average score is $\bar{x} = 2,40$

Worksheet A average score is $\bar{x} = 2,20$

Worksheet A average score is $\bar{x} = 2,40$

Worksheet A average score is $\bar{x} = 2,00$ and Grade 11 general worksheet average score is $\bar{x} = 2,4$ (for the entire worksheet is $2,4 \times 7 = 16,80$)

Participants solved the worksheet A by including two or more geometric thinking habits with the highest percentage of 80%. In C1, C2 and D worksheets the percentage is the same for 3 points in both Grade 10 and 11 grades. In the F worksheet, the percentage is very low for 3 points at both grade levels.

Table 15. Descriptive Analysis of Scores from Geometric Thinking Habits

Variables	N	Lowest	Highest	\bar{x}
10 th Grade the Geometric Thinking Habits score	20	7	21	16.59
11 th Grade the Geometric Thinking Habits score	20	7	21	16.80

Grade 10 students' overall worksheet average score is $\bar{x} = 2,37$ ($2,37 \times 7 = 16,59$). Grade 11 students' general worksheet average score is $\bar{x} = 2,4$ (for the entire worksheet is $2,4 \times 7 = 16,80$). From here, it can be said that the achievements of students' geometric thinking habits are at a medium level.

5. DISCUSSION AND SUGGESTIONS

In the research, platonic objects are included in both high school mathematics curriculum and they are selected to fold the one-dimensional paper and see how it is transported from two dimensions to three dimensions. The aim is to make students fold, acquire geometric concepts, question the properties of objects and make inferences. In addition, worksheets were prepared to investigate the folding process of determining the "geometrical habits of the mind" that will cover four components for the development of geometric thinking. By applying the worksheets prepared, the students were observed throughout the process and the characteristics of the "geometric habits of the mind" were determined. For this reason, the four components developed by Driscoll, Di Matteo, Nikula, and Egan (2007) were determined for the determination of geometric mind habits: reasoning with relationships, investigating invariants, generalizing geometric ideas, and balancing exploration with reflections. In the research, origami-based worksheets were prepared and applied. Using only origami is a limitation of our research. However, it is essential that students learn by doing and living. In this context, the use of origami is suitable for the purpose of the research.

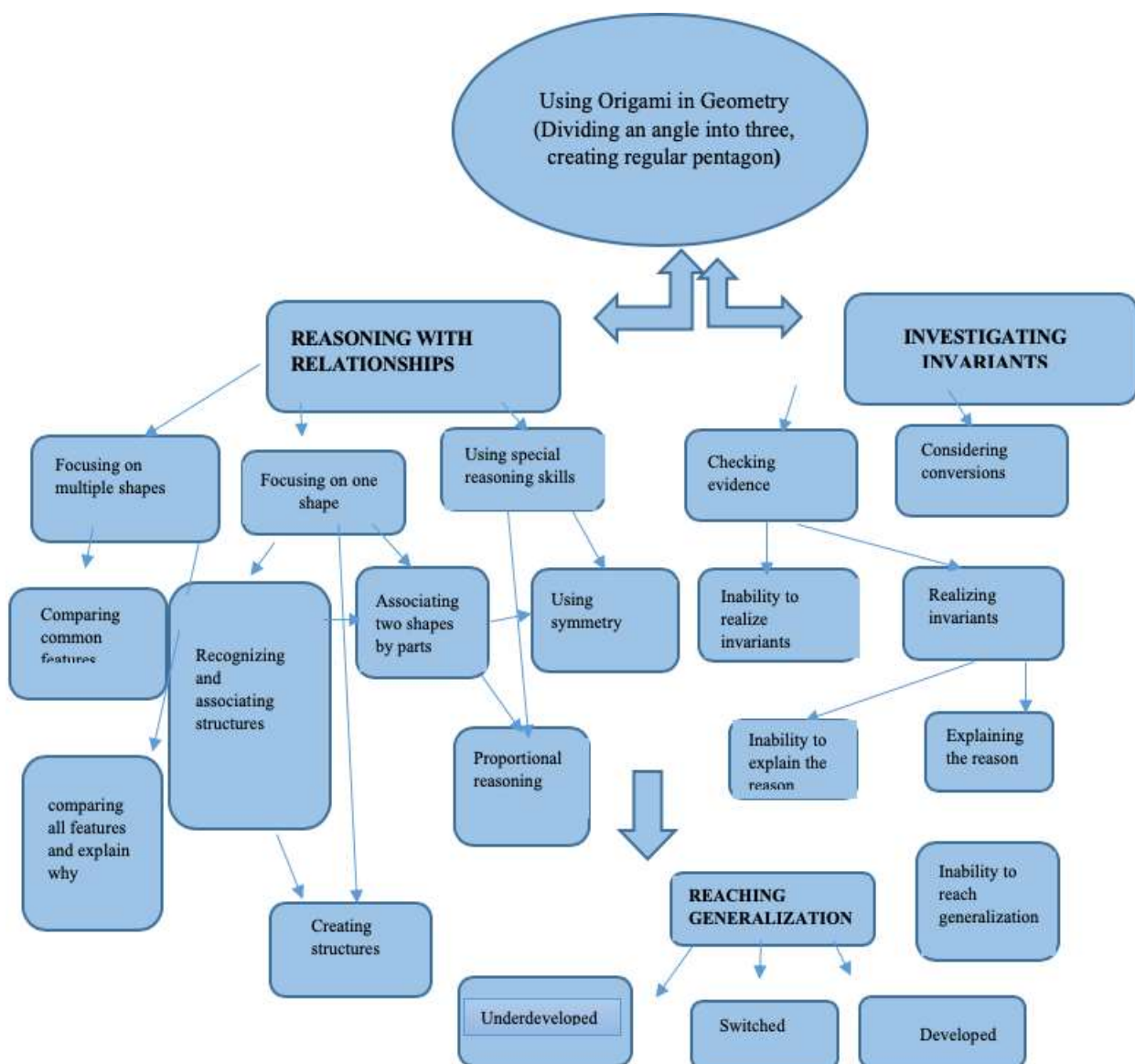


Figure 2. Using Origami in Geometry

As shown in Figure 2, it was seen that by using origami in geometry, the students use more than one geometric habit of the mind skill where the geometric habits of the mind are developed, and that students' problem-solving skills, three-dimensional thinking skills and creativity have improved. As a result of the analysis of students' studies, it was concluded that their skills of reasoning with relationships and investigating invariants improved. As seen in Figure 2, the use of origami in geometry seems to be a necessity. Creating a cube or right prism using origami or krigami from the fifth grade is an activity encountered in mathematics books. However, the activities that are suitable for the creation of tetrahedron,etc., or even with origami are not given enough space.

While creating these objects, it can be classified as a unique activity in the field since it is seen that a math activity is not used in the course with the recognition of the ratio-proportion and polygons. In studies conducted both in our country and abroad, it has been determined that origami contributes significantly to the success of mathematics and geometry while increasing students' reasoning and spatial thinking skills (Boakes, 2009; Arıcı, 2012; Arıcı & Aslan-Tutak, 2013). It can be stated that the exploration and reflection skills of the students are improved and that origami has an effect on it. In the studies of Köse and Tanisli (2014), it is suggested that the findings coincides in that exploration and reflection develop as a result of trying different ways necessary for solution in case of a problem and evaluating the situation at every stage, explaining the mathematical concepts and establishing problems that require them to defend them.

As a result, it was observed that geometric thinking habits worksheets scores were slightly above the middle, since the research was conducted with science high school students. This result is not surprising. When the general exam results are analyzed, the Undergraduate Placement Exam geometry questions part averages are parallel to this result, and in 2014, the average of geometry questions was 5.47; in the following years, respectively: 3,78 and 4,22 (MSPC), 2014; 2015; 2016). These results emphasize the necessity of developing geometric habits. Our research has shown that when we use origami, geometric habits can be developed. This result is similar to the study by Driscoll et al. (2007). In C2, D, E and F, the use of geometry was found to be beneficial in finding relationships, proving and generalizing.

According to the results of the research, some suggestions for the support and development of the Geometric Habits of the Mind are as follows:

- ✓ Studies can be conducted in other high schools in terms of the Geometric Habits of the Mind, generalizing geometric ideas, investigating invariants, balancing exploration and reflection.
- ✓ Appropriate trainings should be provided to both teachers and prospective teachers in terms of using origami in teaching geometry at each stage in order to gain mental skill habits.
- ✓ In this study, the effect of technology on habits was not studied. However, the habitual processes with multiple representations should be examined by making use of the technology in subsequent studies.
- ✓ The relationship between geometric thinking habits and different cognitive or affective dimensions can be examined.

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